

# Mathematics

## **PROGRESSION IN MATHEMATICS**

In mathematics the level descriptions indicate the progression in the knowledge, understanding and skills set out in the different sections of the programmes of study. These are:

### **At key stage 1**

- Number
- Shape, space and measures

### **At key stage 2**

- Number
- Shape, space and measures

### **At key stage 3**

- Number
- Shape, space and measures
- Handling data

The mathematics programmes of study and the National Numeracy Strategy Frameworks for teaching mathematics from reception to year 6 and years 7 to 9 are fully aligned. The Frameworks provide a detailed basis for implementing the statutory requirements of the programme of study for key stages 1, 2, and 3 in mathematics.

Progression by key stage

### **During key stage 1**

Pupils develop their knowledge and understanding of mathematics through practical activity, exploration and discussion. They learn to count, read, write and order numbers to 100 and beyond. They develop a range of mental calculation skills and use these confidently in different settings. They learn about shape and space through practical activities, which build on their understanding of their immediate environment. They begin to grasp mathematical language, using it to talk about their methods and explain their reasoning when solving problems.

### **During key stage 2**

Pupils use the number system more confidently. They move from counting reliably to calculating fluently with all four number operations. They always try to tackle a problem with mental methods before using any other approach. Pupils explore features of shape and space and develop their measuring skills in a range of contexts. They discuss and present their methods and reasoning using a wider range of mathematical language, diagrams and charts.

### **During key stage 3**

Pupils take increasing responsibility for planning and executing their work. They extend their calculating skills to fractions, percentages and decimals, and begin to understand the importance of proportional reasoning. They are beginning to use algebraic techniques and symbols with confidence. They generate and solve simple equations and study linear functions and their corresponding graphs. They begin to use deduction to manipulate algebraic expressions. Pupils progress from a simple understanding of the features of shape and space to using definitions and reasoning to understand geometrical objects. As they encounter simple algebraic and geometric proofs, they begin to understand reasoned arguments. They communicate mathematics in speech and a variety of written forms, explaining their reasoning to others. They study handling

data through practical activities and are introduced to a quantitative approach to probability. Pupils work with increasing confidence and flexibility to solve unfamiliar problems. They develop positive attitudes towards mathematics and increasingly make connections between different aspects of mathematics.

#### **During key stage 4 (foundation)**

Pupils consolidate their understanding of basic mathematics, which will help them to tackle unfamiliar problems in the workplace and everyday life and develop the knowledge and skills they need in the future. They become more fluent in making connections between different areas of mathematics and its application in the world around them. They become increasingly proficient in calculating fractions, percentages and decimals, and use proportional reasoning in simple contexts. Building on their understanding of numbers, they make generalisations using letters, manipulate simple algebraic expressions and apply basic algebraic techniques to solve problems. They extend their use of mathematical vocabulary to talk about numbers and geometrical objects. They begin to understand and follow a short proof, and use geometrical properties to find missing angles and lengths, explaining their reasoning with increasing confidence. They collect data, learn statistical techniques to analyse data and use ICT to present and interpret the results.

#### **During key stage 4 (higher)**

Pupils take increasing responsibility for planning and executing their work. They refine their calculating skills to include powers, roots and numbers expressed in standard form. They learn the importance of precision and rigour in mathematics. They use proportional reasoning with fluency and develop skills of algebraic manipulation and simplification. They extend their knowledge of functions and related graphs and solve a range of equations, including those with non-integer coefficients. They use short chains of deductive reasoning, develop their own proofs, and begin to understand the importance of proof in mathematics. Pupils use definitions and formal reasoning to describe and understand geometrical figures and the logical relationships between them. They learn to handle data through practical activities, using a broader range of skills and techniques, including sampling. Pupils develop the confidence and flexibility to solve unfamiliar problems and to use ICT appropriately. By seeing the importance of mathematics as an analytical tool for solving problems, they learn to appreciate its unique power.

#### **Progression by level**

##### **Level 1**

Typically, pupils:

- represent their work with objects or pictures and discuss it
- recognise and use a simple pattern or relationship
- count, order, add and subtract numbers when solving problems involving up to 10 objects and can read and write the numbers involved
- use everyday language to describe properties and positions
- measure and order objects using direct comparison, and order events
- sort objects and classify them, demonstrating the criterion they have used.

##### **Level 2**

Typically, pupils:

- select the mathematics they use in some classroom activities and discuss their work using mathematical language
- represent work using symbols and simple diagrams

- count sets of objects reliably, and use mental recall of addition and subtraction facts to 10
- use their understanding of place value to order numbers up to 100
- solve addition and subtraction problems
- use mental calculation strategies to solve number problems involving money and measures
- recognise sequences of numbers, including odd and even numbers
- use mathematical names for common 3-D and 2-D shapes and describe their properties
- distinguish between straight and turning movements, understand angle as a measurement of turn, and recognise right angles in turns
- use non-standard and standard units to measure length and mass
- sort objects and classify them using more than one criterion
- record results in simple lists, tables and block graphs, in order to communicate their findings.

### **Level 3**

Typically, pupils:

- try different approaches to problems to overcome difficulties
- organise their work and check results
- discuss their mathematical work and explain their thinking
- use and interpret mathematical symbols and diagrams
- show understanding of place value in numbers up to 1000, use decimal notation and recognise negative numbers, in contexts such as money and temperature
- use mental recall of addition and subtraction facts to 20 in solving problems involving larger numbers
- use mental recall of the 2, 3, 4, 5 and 10 multiplication tables and derive the associated division facts
- solve whole-number problems involving multiplication or division, including those that give rise to remainders
- use simple fractions that are several parts of a whole and recognise when two simple fractions are equivalent
- classify 3-D and 2-D shapes in various ways
- use non-standard units, standard metric units of length, capacity and mass, and standard units of time, in a range of contexts
- extract and interpret information presented in simple tables and lists
- construct and interpret bar charts and pictograms.

### **Level 4**

Typically, pupils:

- are developing strategies for solving problems and present information and results in a clear and organised way
- use their understanding of place value to multiply and divide whole numbers by 10 or 100
- use a range of mental methods of computation with the four operations, including mental recall of multiplication facts up to 10 10 and quick derivation of corresponding division facts
- use efficient written methods of addition and subtraction and of short multiplication and division
- check the reasonableness of their results by reference to their knowledge of the context or to the size of the numbers

- recognise approximate proportions of a whole and use simple fractions and percentages to describe these
- recognise and describe number patterns, and relationships including multiple, factor and square and begin to use simple formulae expressed in words
- use and interpret coordinates in the first quadrant
- make 3-D mathematical models and draw common 2-D shapes in different orientations on grids
- reflect simple shapes in a mirror line
- choose and use appropriate units and instruments, interpreting, with appropriate accuracy, numbers on a range of measuring instruments
- find perimeters of simple shapes and find areas by counting squares
- collect discrete data, group data where appropriate, draw and interpret frequency diagrams and construct and interpret simple line graphs

### **Level 5**

Typically, pupils:

- identify and obtain necessary information to solve problems and check their results
- show understanding of situations by describing them mathematically using symbols, words and diagrams and draw conclusions of their own explaining their reasoning
- use their understanding of place value to multiply and divide whole numbers and decimals
- order, add and subtract negative numbers in context: use all four operations with decimals to two places
- can reduce a fraction to its simplest form and solve simple problems involving ratio and direct proportion. They calculate fractional or percentage parts of quantities and measurements, using a calculator where appropriate
- understand and use appropriate non-calculator methods to solve problems that involve multiplying and dividing any three-digit number by any two-digit number. They check their solutions by applying inverse operations or estimating using approximations
- construct, express in symbolic form, and use simple formulae involving one or two operations and use brackets appropriately
- use and interpret coordinates in all four quadrants
- measure and draw angles to the nearest degree, and use language associated with angle. pupils know the angle sum of a triangle and that of angles at a point
- identify all the symmetries of 2-D shapes
- know the rough metric equivalents of imperial units still in daily use and convert one metric unit to another
- make sensible estimates of a range of measures · understand and use the formula for the area of a rectangle
- understand and use the mean of discrete data and compare two simple distributions, using the range and one of the mode, median or mean
- interpret graphs and diagrams, including pie charts, and draw conclusions
- understand and use the probability scale from 0 to 1.

### **Level 6**

Typically, pupils:

- carry through substantial tasks and solve complex problems by breaking them down into smaller, more manageable tasks

- give mathematical justifications: use trial-and-improvement methods involving approximating and ordering decimals
- can evaluate one number as a fraction or percentage of another
- understand and use the equivalences between fractions, decimals and percentages and calculate using ratios in appropriate situations
- add and subtract fractions by writing them with a common denominator
- can find and describe in words the rule for the next term or nth term of a sequence where the rule is linear
- formulate and solve linear equations with whole-number coefficients
- represent mappings expressed algebraically: recognise and use common 2-D representations of 3-D objects
- know and use the properties of quadrilaterals in classifying different types of quadrilateral
- solve problems using angle and symmetry properties of polygons and angle properties of intersecting and parallel lines, and explain these properties
- devise instructions for a computer to generate and transform shapes and paths
- understand and use appropriate formulae for finding circumferences and areas of circles, areas of plane rectilinear figures and volumes of cuboids when solving problems
- enlarge shapes by a positive whole-number scale factor
- collect and record continuous data
- construct and interpret frequency diagrams, construct pie charts and draw conclusions from scatter diagrams
- use their knowledge that the total probability of all the mutually exclusive outcomes of an experiment is 1.

### **Level 7**

Typically, pupils:

- refine or extend the mathematics used to generate fuller solutions
- justify their generalisations, arguments or solutions, showing some insight into the mathematical structure of the problem
- appreciate the difference between mathematical explanation and experimental evidence
- round to one significant figure and multiply and divide mentally
- understand the effects of multiplying and dividing by numbers between 0 and 1
- solve numerical problems involving multiplication and division with numbers of any size, using a calculator efficiently and appropriately
- understand and use proportional changes
- find and describe in symbols the next term or nth term of a sequence where the rule is quadratic; they multiply two expressions of the form  $(x + n)$  and simplify the corresponding quadratic expressions
- use algebraic and graphical methods to solve simultaneous linear equations in two variables and solve simple inequalities
- understand and apply Pythagoras' theorem when solving problems in two dimensions
- calculate lengths, areas and volumes in plane shapes and right prisms
- enlarge shapes by a fractional scale factor, and appreciate the similarity of the resulting shapes
- determine the locus of an object moving according to a rule
- appreciate the imprecision of measurement

- understand and use compound measures, such as speed
- specify and test hypotheses
- determine the modal class and estimate the mean, median and range of sets of grouped data
- use measures of average and range
- draw a line of best fit on a scatter diagram, by inspection
- understand relative frequency as an estimate of probability and use this to compare outcomes of experiments.

### **Level 8**

Typically, pupils:

- develop and follow alternative approaches, reflecting on their own lines of enquiry
- convey mathematical or statistical meaning through precise and consistent use of symbols that is sustained throughout the work
- solve problems involving calculating with powers, roots and numbers expressed in standard form
- choose to use fractions or percentages to solve problems involving repeated proportional changes or the calculation of the original quantity given the result of a proportional change
- evaluate algebraic formulae, substituting fractions, decimals and negative numbers
- calculate one variable, given the others, in formulae such as  $v = yr^2h$
- manipulate algebraic formulae, equations and expressions, finding common factors and multiplying two linear expressions. They know that  $a^2 - b^2 = (a + b)(a - b)$ . They solve inequalities in two variables
- sketch and interpret graphs of linear, quadratic, cubic and reciprocal functions, and graphs that model real situations
- understand and use congruence and mathematical similarity
- use sine, cosine and tangent in right-angled triangles when solving problems in two dimensions
- distinguish between formulae for perimeter, area and volume, by considering dimensions
- interpret and construct cumulative frequency tables and diagrams, using the upper boundary of the class interval
- estimate the median and inter-quartile range and use these to compare distributions and make inferences
- understand how to calculate the probability of a compound event and use this in solving problems.

### **Exceptional performance**

Typically, pupils:

- give reasons for the choices they make when investigating within mathematics and explain why particular lines of enquiry or procedures are followed and others rejected
- apply the mathematics they know in familiar and unfamiliar contexts
- use mathematical language and symbols effectively in presenting a convincing reasoned argument including mathematical justifications
- understand and use rational and irrational numbers
- determine the bounds of intervals
- understand and use direct and inverse proportion

- use rules of indices for negative and fractional values
- express general laws in symbolic form
- solve simultaneous equations in two variables where one equation is linear and the other is quadratic
- solve problems using intersections and gradients of graphs
- sketch the graphs of sine, cosine and tangent functions for any angle, and generate and interpret graphs based on these functions
- use sine, cosine and tangent of angles of any size, and Pythagoras' theorem when solving problems in two and three dimensions
- use the conditions for congruent triangles in formal geometric proofs
- calculate lengths of circular arcs and areas of sectors, and calculate the surface area of cylinders and volumes of cones and spheres
- appreciate the continuous nature of scales that are used to make measurements
- interpret and construct histograms
- understand how different methods of sampling and different sample sizes may affect the reliability of conclusions drawn
- recognise when and how to work with probabilities associated with independent mutually exclusive events.