

STEP Mathematics

Curriculum Overview 2020-2021

Core aims of the subject at Key Stage 5

“Mathematics expresses values that reflect the cosmos, including orderliness, balance, harmony, logic, and abstract beauty.” **Deepak Chopra**

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history’s most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject.

Our curriculum in mathematics aims to develop fluency in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that students develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately. We are also striving to allow students to reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, develop mathematical arguments and proofs and make conclusions based on logical inferences. Our intention is also for students to solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions; as such resilience is a crucial skill that we will be cultivating in students. Students also need to be able to utilise technology effectively, such as scientific calculators, to perform increasingly complex problems (as well having strong written and mental mathematical skills, not instead of). As the repertoire of mathematical skills that a student possesses grows increasingly more complex, so should the ability of students to use their mathematics to model real life situations.

At key stage 3, we promote equality by working through the breadth of the curriculum at the same pace for all students so that all students can achieve regardless of their starting point. All students at key stage 3 have the option of attending a number of lunchtime clubs in mathematics (Countdown club, 24 club, TT Rockstars club). We extend our highest attaining students through depth and more challenging problem solving, rather than an acceleration of content. At key stage 4, all students continue their mathematical studies on an appropriate GCSE pathway determined by their prior attainment and performance across key stages 2 and 3 to give them the best chance of achieving well in the subject. The GCSE curriculum is also supplemented for the most able students by offering Additional Maths to inspire, motivate and support students in their transition to key stage 5; in all key stages we also enter our most able students into the annual UKMT maths challenge competition. We intend for a high proportion of our students to go on and study or use mathematics in some form post-16; this means that our key stage 4 curriculum needs to be broad enough to cater for students who will go on to study maths at the highest level in our Further Mathematics and STEP preparation classes to those who will study Core Mathematics to complement their A Level choices.

In addition to the obvious intention of preparing students for more advanced mathematical studies, mathematics also prepares students for future learning in other disciplines and improves the cognitive ability in general of students and, as such, is essential for their personal development. The study of mathematics conditions the brain to see connections and builds neural pathways that make the brain stronger for all other things. Mathematical study enhances students' general intelligence and supports the life-long learning of students by: creating a framework in the brain for systematic thinking, developing the ability to solve and analyse problems, stretching the mind to work on unfamiliar tasks with confidence, developing the sequencing skills critical to arriving at accurate results or logical conclusions, promoting caution and care in thinking and deciphering complex mathematical problems to arrive at an accurate answer and learning through trial and error to integrate different principles to arrive at a logical conclusion.

In addition to these disciplinary aspects of the mathematics curriculum, the actual mathematical knowledge and skills that students learn are also vitally important in allowing students to achieve elsewhere in school. Topics studied in mathematics are prerequisite for several disciplines across key stages 3, 4 and 5 such as geography, psychology and economics (to name only a few). Mathematics also provides a theoretical springboard for the ever-evolving STEM sector. Nationally, there are huge shortfalls in job applicants with strong STEM skills and reports estimate that the cost of this shortfall is £1.5 billion each year. Furthermore, occupations in the STEM sector are growing at a rate that nearly is nearly double other sectors which could see this shortfall exacerbated. Our intention is to develop students' abilities sufficiently so that they are able to rise to the challenging opportunities this sector has to offer. In the 21st century science, technology and engineering are constantly changing and have become increasingly important for society. The mathematical principles that govern these areas, however, have not changed and consequently the breadth and depth of our curriculum aims to future-proof our students in this field. This is all in addition to the inspirational and motivating research that claims that candidates with strong maths skills earn on average 11% more in their lifetime.

Mathematics is a discipline which is universal; transcending language and cultural differences. Throughout its rich history, mathematics has adopted elements from around the world and gives students the opportunity to appreciate fundamental truths and create water-tight arguments based on logic and reasoning; as such it helps contribute to the student's spiritual, moral, spiritual and cultural development.

Ultimately, the intention of the maths curriculum is to provide students with the necessary thinking skills and content to be successful in their next stage of life or education.

Assessment

STEP Mathematics is largely self-assessed since it is aiming to develop students' independent learning skills with mathematics and much of the content is either right or wrong. However, there is also a focus on constructing water-tight, logical arguments which will be assessed by the class teacher; due to the nature of course, this usually happens 'live' during the lesson so that students can improve their logic and develop their reasoning skills. With the exception of any mock exams, STEP Mathematics is usually assessed within the lesson itself. At the end of the course students sit either a STEP paper (II or III, STEP I has now been discontinued) or the Advanced Extension Award (AEA) paper. Many students also sit the MAT exam during year 13 as part of their university application which STEP lessons also help to prepare for.

Homework

Students will be encouraged to work on their assignments in their study periods but no formal homework is set for STEP. However, students who require to sit MAT, STEP II or STEP III as part of their university application should expect to put in a lot of work outside of lesson to prepare for these exams.

Clubs and/or intervention

Drop-in sessions run each week
UKMT Mentoring runs each week

Parental/Carer support

Encourage students to attend drop-in sessions

Connections to Learning

Mathematics is a highly inter-connected discipline and studying STEP Mathematics heightens students' appreciation of the links between different topics and deepening understanding of topics and concepts covered in A Level Mathematics and A Level Further Mathematics. In these lessons, students should prioritise their problem solving skills, depth of understanding and their ability to argue rationally above the curriculum content which is already taught within the A Levels.

Connections to Future Pathways

STEP Mathematics is specifically intended to support students progress onto a mathematics (or mathematical) degree at one of the most selective universities in the country. While this is the primary purpose of the qualification, there is merit in studying it to improve understanding of mathematics in general. It is, however, a very specialised course and is ideally suited to students who are going to study Mathematics, Physics, Computer Science or Engineering at university.

Helpful sources of information

<https://www.drfrostmaths.com/> provides a lot of the Powerpoint slides and resources that are used in lessons, this is also the platform where students complete their homework.
<https://www.pearsonactivelearn.com/> has electronic access to the course textbooks (students will need a login from their class teacher to access these)
<https://integralmaths.org/> has a lot of support for students and contains assessments (as well as revision notes) for each unit across years 12 and 13
<https://www.desmos.com/> is a free graphing tool which students will find helpful to aid coordinate geometry problems; there is also an app for this that they can install on their phone
<https://maths.org/step/> provides resources to help students to prepare for STEP
<https://stepdatabase.maths.org/database/index.html> has previous year's STEP questions organised by topic to help students work on key areas
<https://www.admissionstesting.org/for-test-takers/step/preparing-for-step/> Information about the STEP exam

<https://www.maths.ox.ac.uk/study-here/undergraduate-study/maths-admissions-test> Information about the MAT exam

<https://qualifications.pearson.com/en/qualifications/edexcel-a-levels/advanced-extension-award-mathematics-2018.html> Information about the AEA exam

There is not a particular timeline for students to work through the assignments. It is better for students to progress through the assignments at their own pace, ensuring that they have fully understood the problems and taking time to appreciate their techniques and see the connections between the assignments and their A Level content than rush through to keep up with anyone else in the class.

The assignments listed below are Foundation STEP assignments which are given to all students initially. Depending on university offers, some students will deviate from these assignments and focus more on preparation for MAT or STEP II and III.

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| Assignment 1 | <p>The warm up for this assignment involves manipulating surds and simplifying them. The answers to the last part can be expressed surprisingly neatly!</p> <p>The main STEP question (2005 STEP 1 Question 3) is all about the solutions (or roots) of quadratic equations, and whether an equation has 0, 1 or 2 real roots. As is often the case with STEP questions, the coefficients in the equation are not numerical (they are in terms of parameters such as a, b and c).</p> |
| Assignment 2 | <p>The warm up for this assignment involves some algebraic manipulation. There are a few different approaches you can take for each part. The difference of two squares identity might be useful.</p> <p>The main STEP question (1999 STEP 1 Question 6) is all about finding the maximum and minimum values of an expression. It doesn't ask you to sketch any graphs, but doing so will help you understand the problem and see what is happening in the different cases.</p> |
| Assignment 3 | <p>The warm up for this assignment involves the sigma notation, and a proof of the formula for the sum of the terms of a geometric progression. For the last part, use the formula rather than summing the individual terms, and try to do this without using a calculator.</p> <p>The main STEP question (2004 STEP 1 Question 2) introduces the “floor” notation which students are unlikely to have seen before but is commonly used in STEP exams. The final part involves a linear Diophantine equation, i.e. one of the form $ax+by = c$ where a, b and c are given integers and we are looking for a solution where x and y are also integers.</p> |
| Assignment 4 | <p>The warm up for this assignment leads you through a proof that the angle at the centre of a circle is twice the angle at the circumference.</p> <p>The main STEP question (1995 STEP 1 Question 1) starts off with a cubic inequality and then leads into inequalities in two variables on the (x,y) plane. It is a nice example of a STEP questions where each part builds upon the one before.</p> |

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| Assignment 5 | <p>The warm up for this assignment shows you how to obtain a couple of trigonometrical results which you will need for later questions.</p> <p>The main STEP question (2006 STEP 1 Question 8) is about a tetrahedron (triangular based pyramid), and using the volume of the tetrahedron to find another length.</p> |
| Assignment 6 | <p>The warm up for this assignment practices algebraic simplification and simultaneous equation solving.</p> <p>The main STEP question (2005 STEP 1 Question 1) is about arrangements, and carefully considering all the possible options. There are some examples on the last page for those of you who have not met these ideas yet.</p> |
| Assignment 7 | <p>The warm up for this assignment is about sketching graphs, and the key points you need to consider when doing this. You can also use your graphs to solve inequalities.</p> <p>The main STEP question (2002 STEP 1 Question 5) is requires students to think about the roots of a quartic equation in an unusual way.</p> |
| Assignment 8 | <p>The warm up for this assignment proves the AM-GM inequality for 2, 3 and 4 values. The AM-GM inequality states that the arithmetic mean of a set of values is greater than or equal to the geometric mean of those values and is a very useful and powerful result.</p> <p>The main STEP question (2002 STEP 1 Question 1) is about the intersections of a circle and an ellipse; you don't need to know anything about ellipses to tackle the question but students will revisit the ellipse in year 13 Further Maths.</p> |
| Assignment 9 | <p>The warm up for this assignment shows you how to prove some results about triangles.</p> <p>The main STEP question (1993 STEP 1 Question 7) is about the graphs of cubics and how many roots the cubics have.</p> |
| Assignment 10 | <p>The warm up for this assignment derives the trigonometrical identities for $\sin(\alpha \pm \beta)$ and $\cos(\alpha \pm \beta)$.</p> <p>The main STEP question (2005 STEP 2 Question 2) introduces a function which will probably be unfamiliar to students but they should not be put off by this. The question starts by asking you to evaluate the function for some specific values so that you get a feel for what it does</p> |
| Assignment 11 | <p>The warm up for this assignment asks questions about two different functions - the first one is one that will be familiar, but students may not have seen it defined in the same way before.</p> |

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| | <p>The main STEP question (2013 STEP 1 Question 1) consists of three equations to solve. Students are given a substitution for the first equation, doing something similar for the other two would be a good plan. This kind of approach is going to be a common theme in subsequent assignments.</p> |
| Assignment 12 | <p>The warm up for this assignment asks you to show that $n(n+1)$ is always even when n is a positive integer, and similar facts.</p> <p>The main STEP question (2011 STEP 1 Question 12) is about selling raffle tickets and the probability that there will be enough change for everyone.</p> |
| Assignment 13 | <p>The warm up for this assignment introduces the definitions of convex and concave graphs, and also discusses points of inflection.</p> <p>The main STEP question (2012 STEP 1 Question 2) uses some of these ideas to help sketch some quartics, and use these to determine how many solutions there are of some related equations.</p> |
| Assignment 14 | <p>The warm up for this assignment introduces the "difference of two cubes" and "sum of two cubes" factorisations, which (like the difference of two squares factorisation) are well worth knowing and not necessarily covered in A Level lessons.</p> <p>The main STEP question (2010 STEP 2 Question 3) involves Fibonacci numbers and an explicit formula for them (rather than the implicit one $F_{n+1} = F_n + F_{n-1}$).</p> |
| Assignment 15 | <p>The warm up for this assignment practises algebraic manipulation and then introduces the product notation (very similar to sigma notation for summations!)</p> <p>The main STEP question (2006 STEP 2 Question 1) is about sequences, and whether they are periodic or converge.</p> |
| Assignment 16 | <p>The warm up for this assignment has some questions about functions.</p> <p>The main STEP question (2015 STEP 1 Question 2) involves a cubic equation which you can solve by using trigonometric identities.</p> |
| Assignment 17 | <p>The warm up for this assignment introduces Modular Arithmetic and shows how you can use this to find divisibility tests for dividing by 3 and 11.</p> <p>The main STEP question (2003 STEP 1 Question 1) derives the results for the sum of n squares and n cubes which students will have met already in Core Pure 1.</p> |
| Assignment 18 | <p>The warm up for this assignment asks you to sketch some graphs.</p> <p>The main STEP question (2014 STEP 1 Question 3) involves a relationship between two integrals to deepen students' understanding of one of the most demanding aspects of the A Level.</p> |

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| Assignment 19 | <p>The warm up for this assignment derives the small angle approximations for $\sin\theta$, $\cos\theta$ and $\tan\theta$ (which hold when θ is in radians).</p> <p>The main STEP question (2005 STEP 1 Question 6) is about coordinate geometry and describing a path of a point.</p> |
| Assignment 20 | <p>The warm up for this assignment uses differentiation by first principles to derive the derivatives of some well-known functions.</p> <p>The main STEP question (1996 STEP 2 Question 3) is about proof by induction, a Further Mathematics topic which appears on the STEP papers. There is also a second STEP question (2006 STEP 3 Question 8) about induction.</p> |
| Assignment 21 | <p>The warm up for this assignment introduces a couple of new functions which might remind you of some well-known ones.</p> <p>The main STEP question (1999 STEP 1 Question 4) is about sketching regions in the x-y plane described by relationships involving the modulus function.</p> |
| Assignment 22 | <p>The warm up for this assignment derives the product rule for differentiation, and asks you to prove some things about the exponential function from its definition as an infinite series.</p> <p>The main STEP question (2015 STEP 1 Question 1) is about sketching graphs and using your sketches to work out how many solutions there are to a particular equation.</p> |
| Assignment 23 | <p>The warm up for this assignment introduces the chain rule for differentiation, with an explanation of why it works.</p> <p>The main STEP question (2009 STEP 1 Question 8) is about the intersections of lines and circles.</p> |
| Assignment 24 | <p>The warm up for this assignment discusses two more integration techniques.</p> <p>The main STEP question (1998 STEP 2 Question 4) has an integration problem for you to solve.</p> |
| Assignment 25 | <p>The warm up for this assignment introduces integration by substitution and applies this to both indefinite and definite integrals.</p> <p>The main STEP question (1994 STEP 1 Question 8) is also about integration by substitution.</p> |
| Practice Questions | <p>Having completed the assignments, students then work on a variety of questions from pure, mechanics and statistics taken from STEP I, II, III, MAT and AEA to continue deepening understanding and improving problem solving skills.</p> |

